An estimate of the average cosine for the radiance distribution resulting from a point source in the ocean.

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ABSTRACT

We have shown in an earlier paper that by measuring the decay of the irradiance field due to a point source, the absorption coefficient of the medium can be determined. A central parameter in this method is the average cosine of the radiance distribution at each measurement point. From measurements of the Point Spread Function (PSF), an estimate of the average cosine can be determined. However, experimentally the PSF is routinely measured only to 12 degrees. We have also previously shown a method which relates the small angle PSF measurements to an empirically derived analytic formulation of the Beam Spread Function (BSF) which extends to 90 degrees. We will present an independent test of this method, and then use the BSF to determine the average cosine for varying ranges (from the point source) and water properties. In this way we can estimate how rapidly the average cosine varies, and its relative importance in the field measurements of the absorption coefficient with these techniques.

1. INTRODUCTION

One of the most important parameters in describing the propagation of light is the total absorption coefficient of the medium. This parameter is also one of the most difficult to properly measure. In almost every technique, light scattering can interfere with the measurement to some extent. Additionally there are unanswered questions about the variability of the absorption coefficient with changes in the measurement scale, the so-called "scales problem". Often the inherent optical properties of the medium, such as scattering and absorption, are required for prediction of the light field through radiative transfer modeling. In this case a large scale measurement technique is most appropriate. The most common method of measuring absorption on a large scale is through some variation of Gershun's Law.[1] This equation relates the average cosine of the light field and the falloff of scalar or vector irradiance with the absorption coefficient. Most commonly, the light source used, or proposed, is the natural solar illumination. In an earlier paper,[2] we discussed how a point source may be used as a light source and illustrated the differences in the resulting Gershun's Law when the light field exhibits spherical symmetry, rather than plane parallel symmetry as resulting from solar illumination. A key factor in this technique is the average cosine of the light field resulting from the point source. In this paper we will illustrate one method of estimating the average cosine, given small angle measurements of the PSF. With these results we will illustrate the predicted variation of the average cosine

2. BACKGROUND

In a previous paper, we showed that the absorption coefficient can be related to the irradiance attenuation from a point source using Gershun's Law in spherical coordinates. The equation relating these factors is:

$$a(z) = \overline{\mu}_r(z)\left[K_{E_r}(z) - \frac{2}{z}\right]$$

Where $a(z)$ is the absorption coefficient, $K_{E_r}(z)$ is the diffuse attenuation coefficient for vector irradiance from an isotropic source, z is the distance from the source, and $\overline{\mu}_r(z)$ is the radial average cosine:

0-8194-1574-X/94/$6.00

$$\bar{\mu}_r(z) = \frac{E_r(z)}{E_o(z)}.$$

The $2/z$ factor is simply due to the inverse z^2 dependence of the spherically dependent light field. $K_{E_r}(z)$ is defined by:

$$K_{E_r}(z) = -\frac{1}{E_r(z)}\frac{dE_r(z)}{dz}.$$

So if $E_r(z)$ as a function of z is measured, and an estimate (or measurement) of $\bar{\mu}_r(z)$ is available, then $a(z)$ may be determined.

The PSF(θ,z), if known for all angles, can provide $\bar{\mu}_r(z)$:

$$\bar{\mu}_r(z) = \frac{\int PSF(\theta,z)\cos(\theta)d\Omega}{\int PSF(\theta,z)d\Omega}.$$

Where θ is the angle between the point source and incoming radiance (zero for radiance coming from the point source, π when going toward the point source). Unfortunately, because the PSF falls off very rapidly with angle, it is difficult to measure this function at large angles. The method we are currently using to measure the PSF was originally developed by Honey[3] and involves a point source (a flashlamp) and a camera system. Descriptions of the instrument and calibration techniques have been discussed previously.[4] However for this discussion it is relevant that this instrumentation only measures the PSF between 1-200 milliradians. Thus to obtain the PSF over a larger angular range, some numerical method must be used to extrapolate these data. We have described a method[5] which uses an empirically derived equation for the Beam Spread Function (BSF) derived by Duntley and coworkers at the Visibility Lab, Scripps Institution of Oceanography.[6] The empirical relationship of the beam spread vs. angle is given by the expression:

$$\mathrm{BSF}(\theta) = \frac{\mathrm{E}(\theta)}{\mathrm{P}} = \frac{10^{(\mathrm{A-C})}\,\theta^{\mathrm{B}}}{2\,\pi\,z^2 \sin\theta} \qquad (1)$$

where:

$$\mathrm{A}=1.260-0.375\,(c\,z)^{[0.710+0.489(a\,/\,c)]} - [1.378+0.053(c\,/\,a)]\,10^{-cz[0.268+0.083(c\,/\,a)]}$$

$$\mathrm{B}=1-2\,(10^{-\mathrm{D}})$$

$$\mathrm{C}=\frac{1}{3}\{\,[\,(\,\theta\,/\,\mathrm{F}\,)^{3/2}+1]^{2/3}-1\}$$

$$\mathrm{D}=c\,z\,[0.018+0.011\,(c\,/\,a)+0.001725\,c\,z]$$

$$\mathrm{F}=[13.75-0.501c\,/\,a]-[0.626-0.0357\,c\,/\,a]\,c\,z+[0.01258+0.00354\,c\,/\,a](c\,z)^2$$

In these expressions E(θ) is the irradiance measured off-axis, P is the beam power, θ the angle (in degrees) with respect to the unscattered collimated light, c is the beam attenuation coefficient, z the range, and a the absorption coefficient. In the earlier paper we have shown a method to fit experimental ocean measurements with specific values of the a/c and cz parameters. To summarize this method:

1. Profiles of the beam attenuation with depth are used to determine the total optical pathlength, τ, for each PSF measurement.

2. From individual graphs of log(PSF) vs. log(angle[milliradian]) the slope, m, in the region between 4 and 100 milliradian is found. This step depends on the observation that the graph of the log(PSF) vs. log(angle) is almost linear in this region.

At this point one has a relationship for the PSF such that:

$$PSF(\theta) = B_1 \, \theta^{-m}.$$

where m is the slope of log(PSF) vs. log(angle). B_1 is related to the offset in this relationship.

3. Next one uses the observation that there is a regular relationship of m with τ. A relationship of the form $m = A * 10^{(-B\tau)}$ provided a good fit with the empirical equation for a given a/c. Figure 1 uses a new data set, obtained in clear water off of Hawaii, to illustrate the behavior of this function.

4. If one uses the fit from Fig. 1 and plots the resultant B on the curve in Fig. 2 a/c can be obtained. Note that this a/c may not be the real a/c in the water. It is the a/c which can be used to fit the experimental data to the empirical formula. In Fig 2 the values are shown for the calculations along with the value found previously for TOTO (Tongue of the Ocean, Bahamas), the coastal Pacific (PO), Sargasso Sea (SS) and Hawaii.

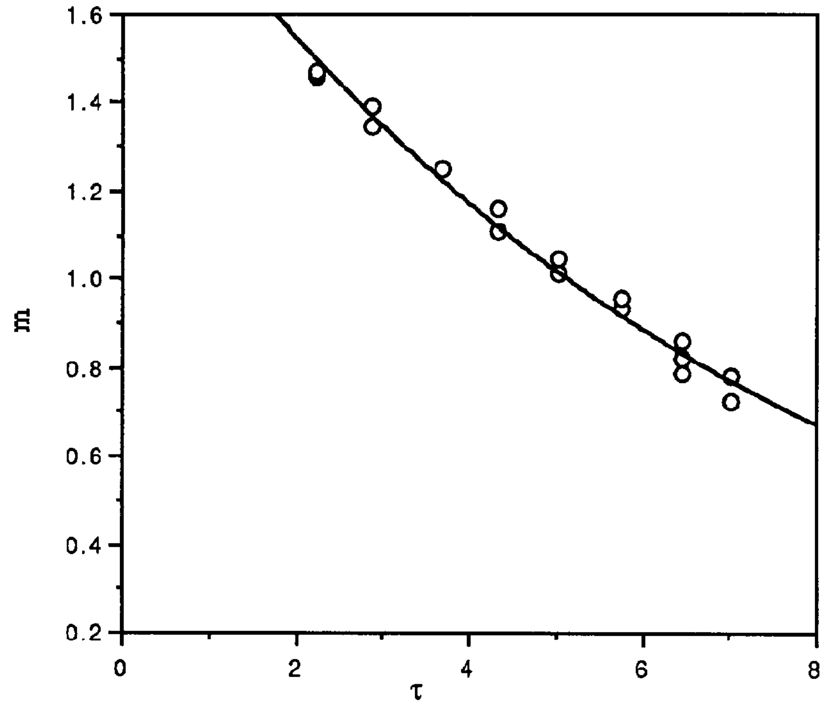

Fig 1) Illustration of functional fit of m vs τ for a data set taken off of Hawaii. This shows how a simple function will relate τ to m for a single data set. All data is from one cast with varying pathlengths in fairly homogenous water. This data was taken at 500 nm.

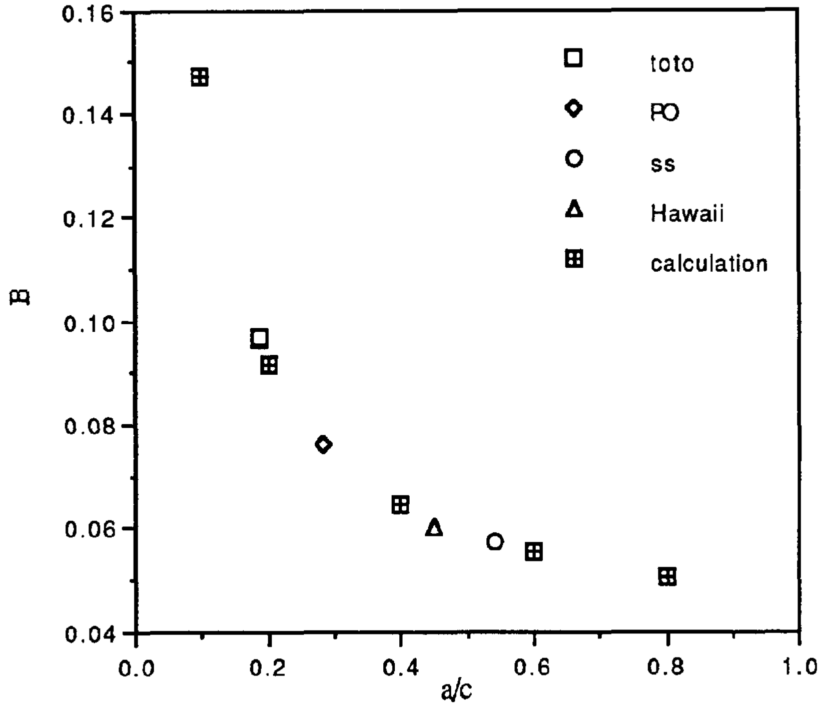

Fig. 2) Relation of a/c for empirical equation and the B coefficient found using m vs t. This graph shows both the results from the empirical equation (B found for each a/c value for the empirical equation) and the B resulting from the different cruise data. This graph enables one to find the correct a/c to use when extrapolating the PSF to larger angles.

5. It is also necessary to make correction to τ using the A derived in the above equation. The correction was empirically found to be:

$$cz = \tau - \log(A/2.158)/B$$

In the case of the Hawaii data this correction results in the addition of 0.41 to the optical pathlength found from the c profiles. This change is probably related to matching the physical size of the source in the oceanic measurements to that of the tank tests. It has a relatively small effect at large ranges, but increases the effective pathlength for short ranges.

After following the above steps, one can generate a PSF using the cz a/c and range (z) with which to compare to our experimental data. Two PSF's from the Hawaii data set, with extreme differences in optical pathlength, were chosentooillustrate the fit. In the short pathlength case τ was 2.23 (obtained from the beam attenuation profile) and the range was 32 m. The parameters needed for the empirical equation were $a/c = 0.45$ and $cz = \tau + 0.41$. The comparison of the experimental PSF and the calculated PSF are shown in Fig. 3. In the long pathlength case τ was 7.07, the range was 100m, and the other factors were constant. The comparison of the experimental data and the empirical equation for this case is shown in Fig 4. In both these cases the PSFs generated with the equation were normalized to the experimental data at 10 milliradians. As can be seen the fit is very good over the range of the experimental data in both cases. The largest deviations appear at small angles in the shorter range data set and is undoubtedly caused by the finite size of the source appearing in our experimental data.

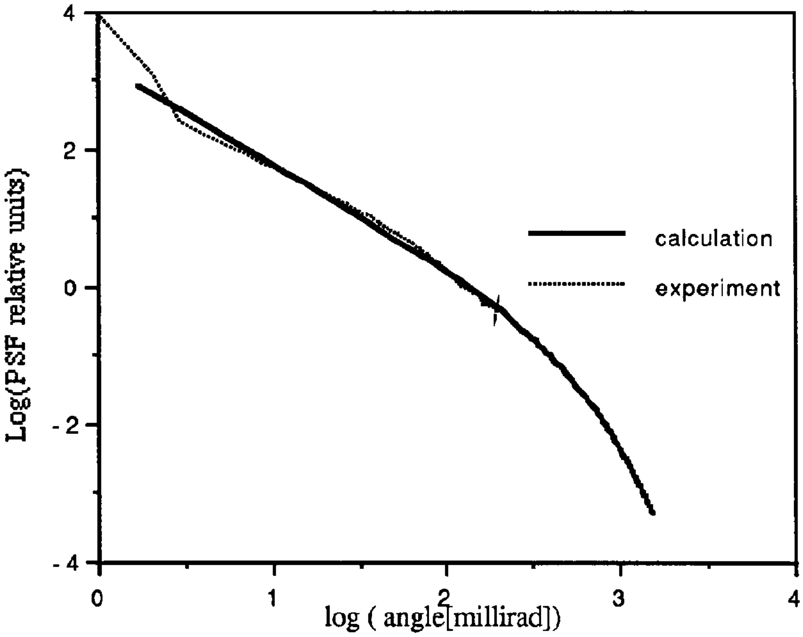

Fig. 3) Graph showing the extrapolated PSF and an experimentally measured PSF. Extrapolation using technique described in text. This data is for 2.23 optical depths, a range of 32 m. a/c = 0.45 and $cz= \tau+0.41$. Data obtained at 500 nm.

3. AVERAGE COSINE

Our method for fitting the measured PSF to Duntley's equation allows us to estimate the average cosine. An important factor is that the extrapolation only extends to 90 degrees. The irradiance reflectance of the water can be used to estimate the error that neglecting the radiance from angles larger than 90 degrees might contribute. For solar illumination, the irradiance reflectance at 500 nm is approximately 5% (+-5%) dependent on the constituents in the water column. Given a water reflectance of this magnitude, the average cosine for 0-90 degrees will overestimate the true average cosine by approximately 10%. This effect will be smallest when close to the source(very peaked radiance distribution) and largest when farther away from the source (more diffuse radiance distribution), and depend on the shape of the volume scattering function of the water.

In Figure 5 we show the variation of the average cosine, calculated with the empirical equation, for the 4 data sets. As can be seen, at short ranges (less than 3 attenuation lengths) the average cosine is larger than 0.9, thus has less of an affect on the absorption measurement. For larger distances however, the average cosine decreases substantially, thus must be taken into account when deriving the absorption coefficient from the irradiance decay. Some of the location dependent variation in the behavior of the average cosine can be removed if, instead of c attenuation lengths, the data is displayed versus b lengths. Since to first order, the width of the PSF (thus the slope, m) should be proportional to the number of scattering events, using b lengths instead of c lengths takes out the dependence on b/c (or a/c). The does remove some of the variations for short attenuation lengths (less than 3 b lengths) however for longer ranges the extra absorption involved in the large angles (longer pathlength) seems to remove the invariance on a. This is an area where we will be doing further work.

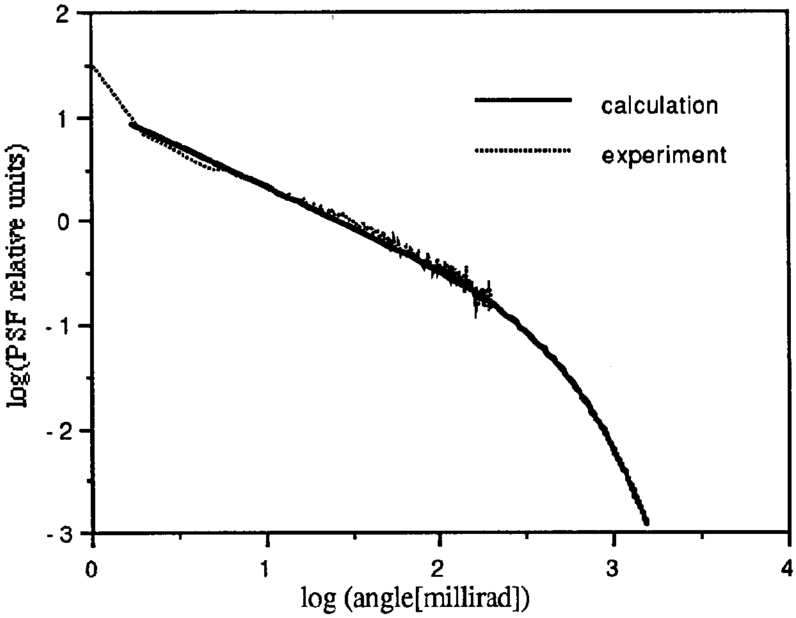

Fig. 4) Graph showing the extrapolated PSF and an experimentally measured PSF. Extrapolation using technique described in text. This data is for 7.07 optical depths, a range of 100 m. $a/c = 0.45$ and $cz= \tau+0.41$. Data obtained at 500 nm.

The effects of the constituent measurements ($\bar{\mu}$, E, E' (the flashlamp irradiance output), and z on the absorption measurement can be found by a simple differentiation of the first equation. After taking the derivative and some substitutions the following equation can be found:

$$\left|\frac{da}{a}\right| = \left|\frac{d\bar{\mu}}{\bar{\mu}}\right| + \left|\frac{dz}{z}\right| + \frac{\bar{\mu}}{az}\left|\frac{dE}{E}\right| + \frac{\bar{\mu}}{az}\left|\frac{dE'}{E'}\right|.$$

Note that the first term on the right is the error in the determination in $\bar{\mu}$ and shows that the error in a is directly proportional to the error in the determination of $\bar{\mu}$ (or ignoring it completely). An error of 20% in $\bar{\mu}$, causes an error in a of 20%. The second error is due to measurement problems with separation of the source and collector. Neglecting problems with wire angle, most of these errors are on the order of 10 cm. At large distances this error term can go away, but it may be important at short distances. The third term is due to error in the measurement of the irradiance, and the fourth term is due to errors caused by the flashlamp variations. Both the third and fourth terms are most important at short distances, and can be minimized in homogenous waters by using least squares line fitting and other techniques which minimize the statistical deviations. Overall the average cosine term is the only term which enters the error budget directly. At short distances this term can be neglected, because the average cosine is very close to 1, however at longer distances, where the other terms are small this term must be taken into account.

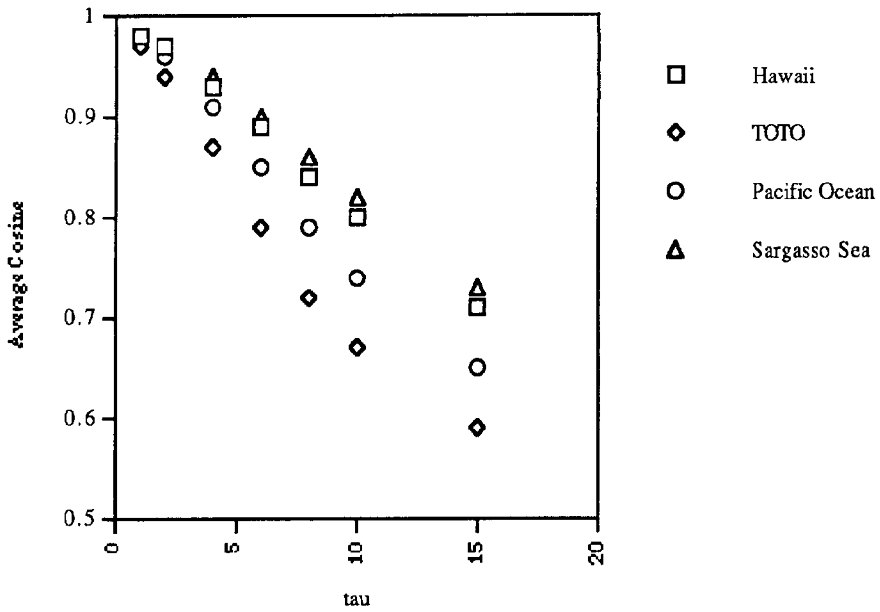

Fig. 5) Average cosine vs tau (τ). Average cosine derived from empirical extrapolation. Coefficients used in the extrapolation derived from the fitting procedure described in text. This is method is based on measurements at for 500 nm. Illustrates that the average cosine can vary significantly in the first 15 attenuation lengths.

4. CONCLUSION

A method first presented in Voss for fitting the measured PSF to Duntley's empirically derived equation for the BSF , was used to extrapolate the measured PSF out to 90 degrees. The extrapolated PSF was then used to estimate the average cosine of the light field due to a cosine source embedded in the ocean. The method appears to work well and reveals a substantial change in the average cosine over a range of 15 optical lengths from the source. When simultaneous measurements of the PSF and the irradiance from an isotropic source are available, our average-cosine estimation technique can be used to improve the accuracy of estimating the absorption coefficient from the irradiance attenuation.

5. ACKNOWLEDGMENTS

This work has been supported by the ocean optics division of the Office of Naval Research. Additional support for field measurements was provided by the Applied Physics Laboratory, Johns Hopkins University. We also thank Albert Chapin for his help in the field measurements.

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