Point spread function in ocean water: comparison between theory and experiment

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A new instrument to measure the point spread function (PSF) in the ocean has provided the opportunity for direct comparison between theoretical predictions and experimental measurement. Theoretical predictions are derived from small angle scattering theory using a simple algebraic fit to the single scattering phase function. The resulting predictions for the PSF are found to match the experimental measurements over a wide range of angles and optical depths. *Key words:* Ocean optics, point spread function, light scattering.

I. Introduction

The point spread function (PSF) characterizes the spatial spreading of optical radiation as it propagates through a scattering media. Such a characterization can be used to quantify the spatial resolution of imaging systems in ocean water and to quantify the spreading of a beam of light as it propagates through the ocean. Thus accurate determination of the PSF is fundamental to quantifying the performance of underwater sensing systems.

Recently, instrumentation has been developed and deployed to measure directly the PSF in ocean water.[1,2] This sensor is capable of measuring the PSF as a function of range (source/detector separation) and assessing the spatial variability of the PSF at a fixed range.

On the theoretical side, the radiative transfer equation (and approximations thereto) provides a link between the single scattering phase function and the integrated effects of multiple scattering. In ocean water, the scattering phase function is very strongly peaked in the forward direction,[3] and small angle approximations to the radiative transfer equation have been found to be remarkably robust.[4]

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Validation of small angle scattering theory has been performed by comparison to Monte Carlo simulations,[4] and the form of the spatial frequency decay function has previously been validated against experimental data.[5] To our knowledge, the results described below provide the first direct comparison of PSF measurements and model predictions and demonstrate the robustness of small angle scattering theory for quantifying the effects of multiple scattering in ocean water.

II. Measurements

The PSF may be defined as the apparent radiance of an unresolved Lambertian source, normalized by the source intensity.[6] Intuitively, the PSF can be thought of as the blurred image of a point source at a given range. This definition of the PSF immediately suggests a method for direct measurement. The measurement technique, first developed by Honey,[7] involves photographing a point source (flashlamp) with an imaging camera system.

The current implementation consists of a xenon flashlamp and a cooled CCD camera with associated control and data acquisition electronics. The source and detector are enclosed in watertight housings, and the separation may be varied to investigate the dependence of the PSF on range. Alternatively, the system may be deployed at a fixed range and used to sample the water column to determine the variability of the PSF in time and space. Details of the sensor fabrication, calibration, and deployment may be found in Ref. 1.

Data have thus far been obtained from three sites: the Tongue of the Ocean (TOTO) Bahamas, the Sargasso Sea, and the coastal Pacific Ocean. The optical environment at the TOTO was found to be reasonably homogeneous and was selected for detailed comparison with theory. This data set includes source/camera separations between 10 and 80 m to evaluate the range dependence of the PSF. The water was also profiled at fixed source/camera separation, and the overall variability with depth was found to be of the order of 20% (averaged over the angle). At this site, the water was quite clear (Jerlov 1B).

III. Theory

Small angle scattering theory is a linearization of the time-dependent radiative transfer equation obtained by approximating $\sin\theta$ by θ and $\cos\theta$ by 1, where θ is the displacement from the unperturbed beam axis. With this approximation, the temporal dependence may be separated out, yielding a linear integro-differential equation describing the lateral spreading of the beam as it propagates through the media. This equation may be solved by integral transform techniques.[8]

In the following, we adopt the formulation given by Wells.[9] The point spread function $f(\theta,R)$ represents the power distribution at range R per unit solid angle displaced by angle θ off-axis, normalized by the power transmitted. The PSF may be related to the modulation transfer function $F(\psi,R)$ through a Hankel transform:

$$F(\psi,R) = 2\pi \int J_0(2\pi\theta\psi)f(\theta,R)\theta d\theta,$$
$$F(\theta,R) = 2\pi \int J_0(2\pi\theta\psi)F(\psi,R)\psi d\psi, \tag{1}$$

The key to the small angle approximation is the relationship between the MTF and the inherent optical properties of scattering and absorption. By considering the effects of infinitesimal slabs of scattering media arranged between the source and receiver, Wells derived

$$F(\psi,R) = \exp\left[-\int_0^R c(r)dr + \int_0^R \sum\left(\frac{\psi r}{R}, r\right)dr\right], \tag{2}$$

where $c(r)$ is the beam attenuation profile, and $\sum(\psi,r)$ is the Hankel transform of the volume scattering function.

While the theory is directly applicable to stratified media, we restrict ourselves here to homogeneous media to avoid the additional complexities of the range integration in Eq. (2). In this case, the scattering term may be written as

$$\sum(\psi,r) = b \cdot S(\psi), \tag{3}$$

where b is the total scattering coefficient and $S(\psi)$ is the Hankel transform of the single scattering phase function $s(\theta)$:

$$S(\psi) = 2\pi \int J_0(2\pi\theta\psi)s(\theta)\theta d\theta. \tag{4}$$

In this case, Eq. (2) for the MTF simplifies to

$$F(\psi,R) = \exp\left[-cR + b\int_0^R S\left(\frac{\psi r}{R}\right)dr\right]. \tag{5}$$

The MTF is thus linked with the inherent optical properties $c,b,s(\theta)$ through Eqs. (4) and (5).

Wells noted that by judicious choice of the scattering phase function,

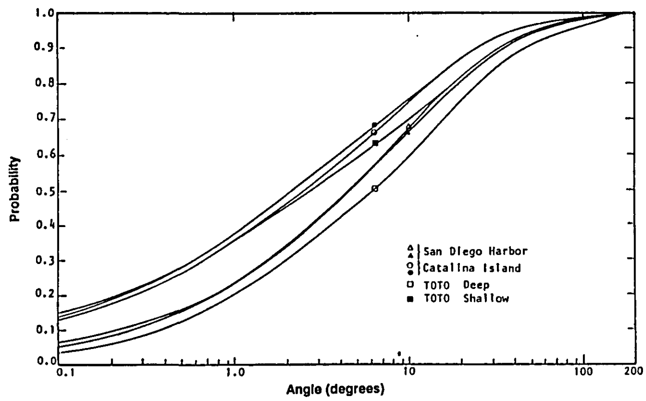


Fig. 1. Single scattering distribution functions for various water types (from Petzold[3]).

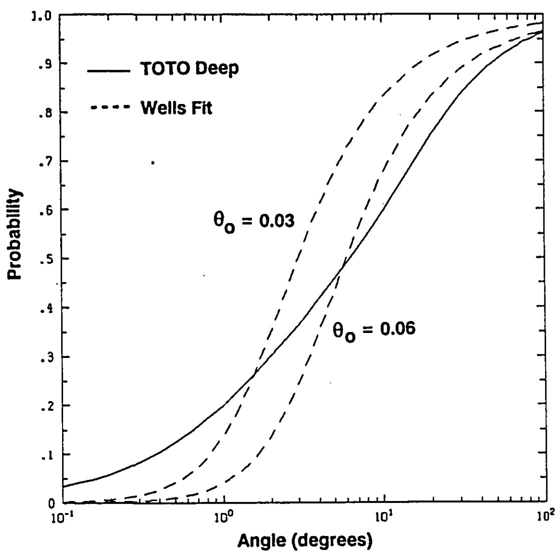


Fig. 2. Comparison of TOTO measurements of the scattering phase function with Wells's algebraic fit. For deep TOTO waters, Wells's model agrees with the median scattering angle if $\theta_0 = 0.06$ rad. Data are from Petzold.[3]

$$s(\theta) = \frac{\theta_0}{2\pi(\theta_0^2 + \theta^2)^{3/2}}, \tag{6}$$

Eqs. (4) and (5) can be integrated explicitly to yield

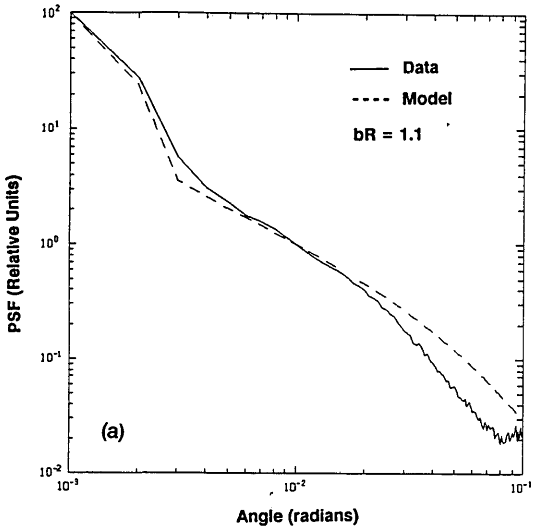
$$S(\psi) = \exp(-2\pi\theta_0\psi), \tag{7}$$

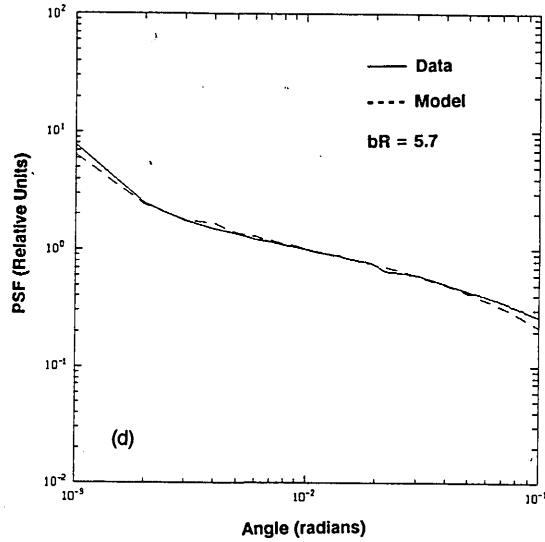
$$F(\psi,R) = \exp\left\{-cR + bR\left[\frac{1-\exp(-2\pi\theta_0\psi)}{2\pi\theta_0\psi}\right]\right\}, \tag{8}$$

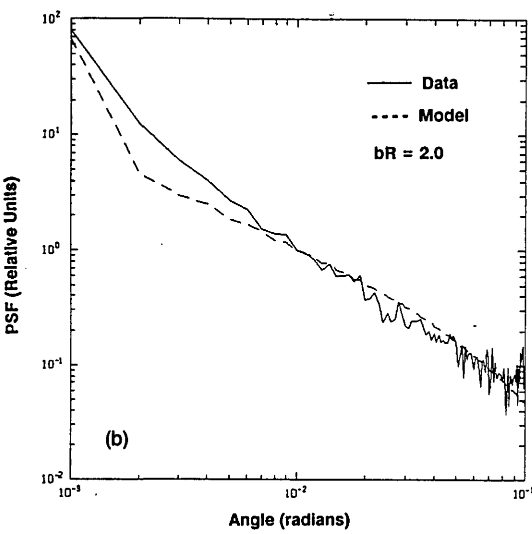
where the parameter θ_0 can be related to the median scattering angle for single scattering. The PSF is then obtained from the MTF by numerical integration of Eq. (1). For a general scattering phase function, Eqs. (4) and (5) cannot be evaluated explicitly, and numerical integration must be utilized for these as well.

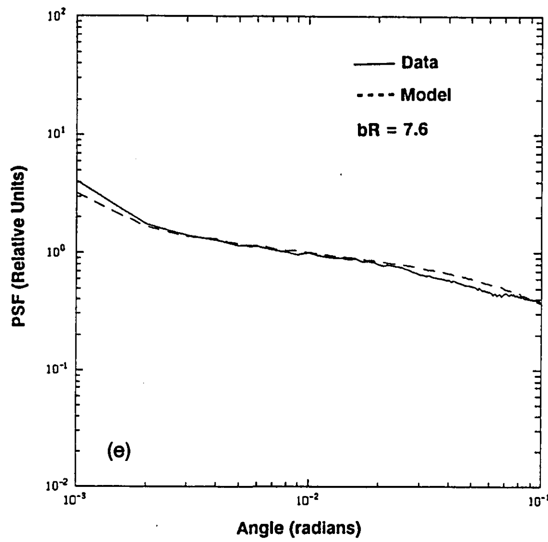
IV. Results

Application of the theory requires specification of the beam attenuation coefficient c, the total scattering









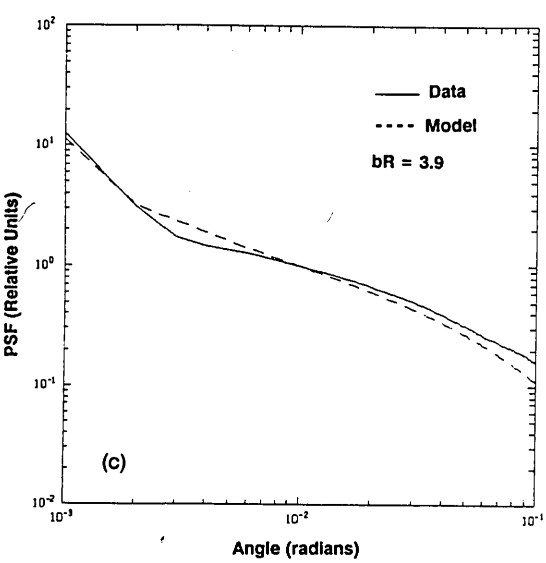


Fig. 3. Comparison of the PSF predicted by small angle scattering theory and TOTO measurements. The predicted PSF has been corrected for the finite sensor resolution: (a) $bR = 1.1$ (11 m); (b) $bR = 2.0$ (20 m); (c) $bR = 3.9$ (39 m); (d) $bR = 5.7$ (57 m); and (e) $bR = 7.6$ (76 m).

coefficient b, and the scattering phase function. For the TOTO data set, beam and diffuse attenuation coefficients were measured to be 0.13/m and 0.045/m, respectively (at 490 nm). Using the Wilson-Honey relationship[10] $b = 6/5\ (c - k)$, we derive $b = 0.10$/m for the total scattering coefficient.

Using Wells's algebraic fit to the scattering phase function, the PSF was obtained by numerical integration of Eq. (1). Initial comparisons with PSF data proved disappointing using the value of $\theta_0 = 0.03$ rad suggested by Wells. To sort out the discrepancy, the algebraic fit to the phase function was compared with the experimental measurements of Petzold.[3] Petzold's measurements (Fig. 1) illustrate that the scattering phase function is dependent on water type and generally exhibits a larger median scattering angle for clear water. The phase function is a weighted combination of molecular scattering which is relatively isotropic and particulate scattering which is strongly peaked. The scattering function for clear water has a relatively large contribution from molecular scattering and thus has a larger median scattering angle.

The algebraic fit to the phase function is chosen for analytical convenience but exhibits considerable discrepancy with Petzold's data at small scattering angles (Fig. 2). To exploit this convenience while attempting to model more faithfully the measured phase function, θ_0 is chosen so that the median scattering angle agrees with the data. For deep AUTEC waters appropriate to the TOTO data, the median scattering angle is 6°, corresponding to $\theta_0 = 0.06$ rad.

Comparison of the PSF between theory and experiment is given in Fig. 3 for five values of range. As the PSF data are known only in relative units, both the data and predictions are normalized to unity at an angle of 10 mrad. Corrections have been applied to account for the finite size of the source (55 mm in diameter) and the finite resolution of the receiver ($\sim$0.5 mrad) by convolving the PSF with a Gaussian profile of appropriate divergence (see Wells[11]). This finite sensor resolution effects the PSF only for angles smaller than $\sim$2 mrad.

The agreement between theory and experiment in Fig. 3 is quite good at all ranges. The greatest discrepancy appears at short ranges ($bR \sim 1$), where the spreading is dominated by single scatter, and the results are most sensitive to the structure of the single scattering phase function. For scattering ranges of 2 and greater, the results appear to be insensitive to the details of the scattering phase function due to multiple scattering effects. Discrepancies at small angles (<5 mrad) at $bR = 2$ are attributed to the finite size of the transmitter. At the larger ranges ($bR = 5.7, 7.6$), experimental data and model predictions are virtually indistinguishable.

These comparisons illustrate that with the appropriate choice of the median scattering angle, small angle scattering theory provides a faithful representation of beam spread for a wide range of radiance values (nearly four decades), angles (two decades), and scattering ranges (nearly one decade). The agreement is particularly remarkable given the obvious discrepancy in the details of the single scattering phase function used in the model. Although it would be possible to use more accurate descriptions of the phase function, there is little motivation to pursue such an approach given the favorable results using this simple model and the analytical simplification resulting from the algebraic fit.

V. Conclusions

We have compared the results of small angle scattering theory with experimental measurements of the PSF. By matching the median scattering angle, the algebraic fit to the scattering phase function is found to yield an accurate prediction of the PSF despite substantial differences in the form of phase function, particularly for $bR > 2$. Thus it appears that the PSF at longer ranges is rather insensitive to the details of the phase function and is controlled by the attenuation coefficient, scattering coefficient, and median scattering angle (for single scatter).

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